DR. A. FERRARI AND PROF L. LALOUI

FORMULAE – LECTURE 3

Triaxial elastic stress-strain relations

$$p=\frac{(\sigma_1+2\sigma_3)}{3} \qquad q=\sigma_1-\sigma_3$$

$$p'=p-p_w \qquad q'=q$$

$$\tau=\frac{\sigma_1-\sigma_3}{2}$$

$$\varepsilon_v=\varepsilon_1+2\varepsilon_3=\frac{1}{K}p' \qquad \varepsilon_d=\frac{2(\varepsilon_1-\varepsilon_3)}{3}=\frac{1}{3G}q$$

Spherical and deviatoric stress tensor decomposition

$$\sigma_{ij}=s_{ij}+p\delta_{ij} \qquad p=\frac{1}{3}\sigma_{nn}$$

General effective stress-strain equation

$$d\sigma'_{ij}=C_{ijkl}d\varepsilon_{kl}$$

Work

$$\delta W=\sigma'_{xx}\delta\varepsilon_{xx}+\sigma'_{yy}\delta\varepsilon_{yy}+\sigma'_{zz}\delta\varepsilon_{zz}+\sigma'_{xy}\delta\varepsilon_{xy}+\sigma'_{yz}\delta\varepsilon_{yz}+\sigma'_{zx}\delta\varepsilon_{zx}$$

$$\delta W=\sigma'_1\delta\varepsilon_1+\sigma'_2\delta\varepsilon_2+\sigma'_3\delta\varepsilon_3=\sigma'_a\delta\varepsilon_a+2\sigma'_r\delta\varepsilon_r$$

$$\delta W=\frac{(\sigma'_a+2\sigma'_r)(\delta\varepsilon_a+2\delta\varepsilon_r)}{3}+\frac{2(\sigma'_a-\sigma'_r)(\delta\varepsilon_a-\delta\varepsilon_r)}{3}$$

$$\delta W=p'\delta\varepsilon_p+q\delta\varepsilon_q=\delta W_v+\delta W_d$$

Isotropic linear elasticity

$$\sigma_{ij}=\left(K-\frac{2G}{3}\right)\varepsilon_{kk}\delta_{ij}+2G\varepsilon_{ij} \qquad \lambda=\frac{E\nu}{(1+\nu)\cdot(1-2\nu)}=K-\frac{2G}{3}$$

$$E=\frac{9KG}{3K+G} \qquad \nu=\frac{3K-2G}{6K+2G} \qquad \varepsilon_{xx}=\frac{1}{E}\left[\sigma_{xx}-\nu\left(\sigma_{yy}+\sigma_{zz}\right)\right]$$

$$K=\frac{E}{3(1-2\nu)} \qquad \tau_{xy}=G\gamma_{xy} \qquad \varepsilon_{yy}=\frac{1}{E}\left[\sigma_{yy}-\nu(\sigma_{zz}+\sigma_{xx})\right]$$

$$\mu=G=\frac{E}{2(1+\nu)} \qquad \varepsilon_{zz}=\frac{1}{E}\left[\sigma_{zz}-\nu\left(\sigma_{xx}+\sigma_{yy}\right)\right]$$

Non-linear elasticity - some models examples

$$\bar{\sigma}=\frac{\bar{\varepsilon}}{a_1+b_1\bar{\varepsilon}} \qquad E_t=\frac{d\bar{\sigma}}{d\bar{\varepsilon}}=\frac{a_1}{(a_1+b_1\bar{\varepsilon})^2} \qquad E_i=E_0\cdot\left(\frac{p}{p_0}\right)^n$$

Wave propagation

$$V_s=\sqrt{\frac{G}{\rho}} \qquad V_p=\sqrt{\frac{\lambda+2G}{\rho}}=\sqrt{\frac{K+\frac{4}{3}G}{\rho}}$$